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THERMAL ANALYSIS OF RESIDENTIAL ELECTRICAL CABLES

Report No 85-272-K

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Mechanical Research Department

ABSTRACT

A steady-state heat transfer analysis has been conducted to study the thermal characteristics of electrical cables embedded in thermal insulation in the attic space of residential homes for various conditions. The analysis was carried out for two cable arrangements and two attic temperatures using the finite element code ABAQUS. The results of the analysis include temperature distributions throughout the modelled regions including the cable. A comparison of the theoretical predictions with existing experimental results shows good agreement. An equation describing the maximum conductor temperature as a function of current and insulation thickness was developed by fitting curves through the finite element results.

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EXECUTIVE SUMMARY

THERMAL ANALYSIS OF RESIDENTIAL ELECTRICAL CABLES

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This report describes the steady-state heat transfer analysis of electrical cables embedded in thermal insulation found in the attic space of residential homes. Two cable configuration were analyzed using the finite element computer code ABAQUS. The results of this study include:

1. Temperature distributions throughout the modelled regions.

2. Equations describing the maximum conductor temperature as a function of current and insulation thickness.

The results of this analysis indicate that the current has a stronger effect on the conductor temperature than the insulation thickness.

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LIST OF SYMBOLS

Variables

h	heat transfer coefficient for natural convection, $W/(mm^2 \cdot °C)$
i	conductor current, amps
k	thermal conductivity, $W/(mm \cdot °C)$
$\dot{q}_B$	heat flux to the junction box due to lightbulb, W/mm^2
$\dot{q}_C$	heat flux to the junction box due to feed lines, W/mm^2
$\dot{q}_J$	total heat flux to the junction box, W/mm^2
t	insulation thickness, mm
T_{AM}	ambient temperature, °C
T_{AT}	attic temperature, °C
T_C	conductor temperature, °C

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To Dr. O.A. Kupcis
Manager
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THERMAL ANALYSIS OF RESIDENTIAL ELECTRICAL CABLES

1.0 INTRODUCTION

The increase in fuel cost in the past decade has led to a greater utilization of insulation by the owners of residential dwellings. However, along with the evident decrease in the heating expenses, a concern has arisen for potential fire hazards due to the increased cable temperature. As a part of a CEA project on "Effects of Thermal Envelope Upgrading on Residential Wiring" undertaken by the Chemical Research Department, heat transfer studies based on finite element methods (see Appendix A) were requested. The effects of insulation thickness and applied current on the temperature of a single electrical cable have been previously investigated both theoretically and experimentally/1,2/. The present study deals with the theoretical heat transfer analysis of the experimental setup shown in Figure 1. The objective is to determine the steady-state temperature distributions including the maximum conductor temperature of a NMD-3 cable (see Figure 2) for the following two configurations using the finite element method:

i) Two cables parallel to each other (see Figure 1, Section A-A).

ii) One cable near a junction box (see Figure 1, Section B-B).

This report describes the finite element models developed for the above configurations, presents the results of the analysis and summarizes the major findings of the study.

2.0 FINITE ELEMENT ANALYSIS

This section describes the two configurations analyzed along with the boundary conditions and presents the finite element models.

2.1 Finite Element Model for Two Cables

Figure 3 shows a cross-section of the two parallel cables and the applied boundary conditions. Since the cables are very long in comparison to their cross-section dimensions, it was assumed that axial heat conduction can be ignored and thus the problem was viewed as two-dimensional.

To reduce modelling effort and computer cost, use of the symmetry conditions has been made. The region shown in Figure 3 was modelled using quadrilateral finite elements. The resulting finite element mesh is shown in Figure 4. The heat generation within the conductors was determined from Joule's law of heating/2/.

2.2 Finite Element Model for One Cable

Figure 5 shows a single cable leading to the inner side of a junction box. As for the two cable configuration, use of the symmetry condition has been made thus reducing model preparation time and computer cost. The finite element model representing this region is shown in Figure 6.

In addition to the internal heat generation in the cable a heat flux, due to the presence of a light bulb and feed wires, had to be applied over the surface area of the junction box. This is described in Appendix B.

3.0 FINITE ELEMENT RESULTS

The temperature results obtained for the finite element models described in Section 2.0 using the computer code ABAQUS are presented and discussed below.

3.1 Temperature Results for Two Cables

Table 2 gives a list of the cases analyzed for this configuration. The temperature results for these cases are presented in graphical form in Figures 7 and 8. Figure 7 shows the conductor temperature as a function of current with insulation thickness and attic temperature as parameters based on an ambient temperature of 25° C. It is evident from this figure that the rate of change of conductor temperature with current is greater for larger insulation thicknesses. The rate

of increase in temperature with current, however, is independent of the attic temperature as one would expect. The same plot also shows the experimental results for insulation thickness of 284 mm and 25° C attic temperature. These results are in good agreement with the theoretical predictions, thus confirming the adequacy of the model used in the analysis. Figure 8 depicts the variation of conductor temperature as a function of insulation thickness with the current as a parameter. Since only two thicknesses were studied, only a general trend can be defined.

Using the least square method of curve fitting and the finite element results, an equation describing the conductor temperature as a function of current and insulation thickness has been developed for two attic temperatures. This equation has the following form:

$$T_C = a_0(t) + a_1(t)i + a_2(t)i^2 + a_3(t)i^3$$

where:

T_C = conductor temperature, °C

i = conductor current, amps

t = insulation thickness, mm

a's = coefficients defining the polynomial

For an attic temperature of 25° C the coefficients of the polynomial are linear functions of the insulation thickness and are given below:

$$a_0(t) = 0.041692t + 13.184$$
$$a_1(t) = -0.0268t + 7.6399$$
$$a_2(t) = 0.00337t - 0.8441$$
$$a_3(t) = -0.0000868t + 0.027778$$

The coefficients for an attic temperature of 50° C correspond to one insulation thickness of 284 mm and are as follows:

$$a_0 = 30.508860$$
$$a_1 = -0.476405$$
$$a_2 = 0.155843$$
$$a_3 = 0.206681 \times 10^{-2}$$

As can be seen from Figure 7, the resulting polynomial adequately represents the finite element results and can thus be used to predict the conductor temperature given the values of the other parameters.

3.2 Temperature Results for One Cable

Only two cases were analyzed for this configuration and they are listed in Table 3. The first case was concerned with the determination of the heat flux to the junction box due to the light bulb. This was accomplished by trying several heat fluxes until the numerical results for T_4 (see Figure 5) matched the experimental results at the same location for a current of 2.7 amps.

Using the contribution of the heat flux from the light bulb at 2.7 amps, the total heat flux to the junction box at 20 amps was calculated as discussed in Appendix B. The predicted temperature at T_4 for the second case compared well with the experimentally measured value.

Figures 9 and 10 show the variation of conductor temperature in the axial direction for currents of 2.7 and 20 amps respectively. These figures indicate that the temperature distributions are practically uniform with very little variation along the conductor.

4.0 CONCLUSIONS

i) Based on the results for the two-cable configuration, the maximum conductor temperature was 100.2° C corresponding to 20 amps and an attic temperature of 50° C. For the case of 25° C attic temperature, this maximum temperature was 92.7° C. Also, the rate of increase in conductor temperature with current is independent of the attic temperature.

ii) A comparison of theoretical predictions with experimental measurements shows good agreement, thus confirming the adequacy of the finite element models.

iii) The equations developed through curve fitting of the finite element results describe the conductor temperature as a function of current for the two insulation thicknesses quite well.

iv) Based on the results for one-cable configuration, the axial variation in conductor temperature is very small. Also, for the same configuration, the major contribution to the total heat flux at the junction box is from the light bulb particularly at low currents. The contribution from the feed lines at low currents is small, but it becomes significant with increasing current.

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2. Taralis, D., "Heat Transfer Analysis of a Residential Electrical Cable Using the Finite Element Method", Ontario Hydro Research Division Report No 80-415-K, October 1980.

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APPENDIX A

A BRIEF DESCRIPTION OF THE FINITE ELEMENT METHOD

The finite element method is a technique of solving differential equations. A continuous problem described by a differential equation is put into an equivalent variational form and the approximate solution is assumed to be a linear combination, $\sum c_j \phi_j$, of approximation functions ϕ_j. The parameters c_j are determined using the associated variational form. The finite element method provides a systematic technique for deriving the approximation functions for simple subregions by which a geometrically complex region can be represented. In the present study the finite element method was used to solve the steady-state heat conduction equation including heat generation subject to appropriate boundary conditions. A number of standard computer programs have been developed based on this method and one such a program (ABAQUS) has been employed to carry out the analysis.

APPENDIX B

DETERMINATION OF HEAT FLUX AT THE JUNCTION BOX

To carry out the heat transfer analysis for one cable, it was necessary to specify a heat flux over the surface area of the junction box. This was determined by assuming the following expression:

$$\dot{q}_J = \dot{q}_B + \dot{q}_C \qquad \text{(B-1)}$$

where:

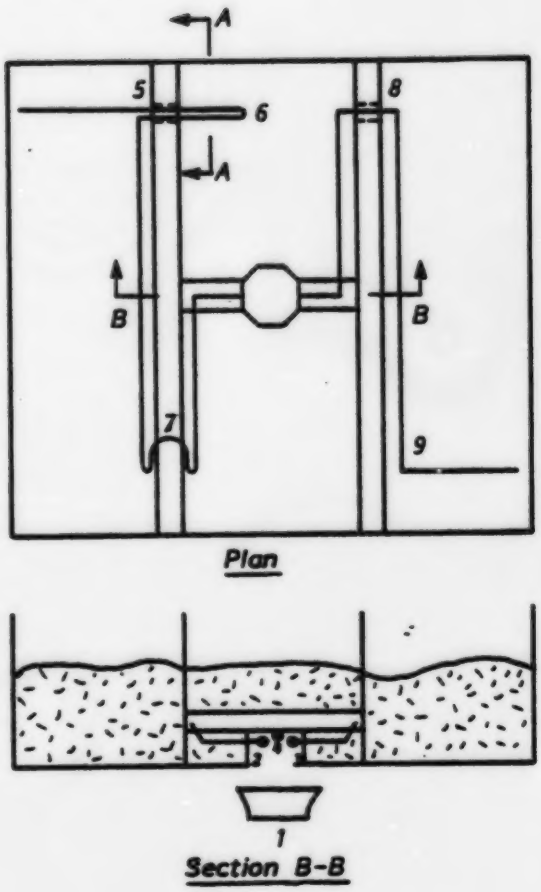
$\dot{q}_J$ = total heat flux to the junction box

$\dot{q}_B$ = heat flux to the junction box due to light bulb

$\dot{q}_C$ = heat flux to the junction box due to the feed lines

The total heat flux to the junction box at low currents was assumed to be approximately equal to the heat flux received from the light bulb ($\dot{q}_C$ is negligible at low currents). Therefore, this flux was determined to be 0.78×10^{-4} W/mm^2 by matching the theoretical results with the experimental results of reference/3/ for i=2.7 amps.

Now to obtain theoretical results at i=20 amps, it is only necessary to calculate $\dot{q}_C$ since $\dot{q}_B$ remains constant. Using the Joule's law of heating/2/ and feed line length of 152.4 mm and surface area of junction box of 20,258 mm^2, the value of $\dot{q}_C$ at 20 amps was calculated to be 0.28×10^{-4} W/mm^2. Therefore, the analysis for a current of 20 amps was carried out by applying a total heat flux of 1.06×10^{-4} W/mm^2 ($0.78 \times 10^{-4} + 0.28 \times 10^{-4}$) at the junction box.



Thermocouple Locations

1. *Top of light fixture*
2. *Feed-in line wire insulation (3 cm from Marr connector)*
3. *Feed-out line wire insulation (3 cm from Marr connector)*
4. *Inner side of junction box centre*
5. *Feed-in line wire insulation (in hole of stud)*
6. *Feed-in line wire insulation (on loop 15 cm from stud)*
7. *Feed-in line wire insulation (on loop over stud)*
8. *Feed-out line wire insulation (in hole of stud)*
9. *Feed-out line wire insulation (on stud)*

FIGURE 1

SCHEMATIC DIAGRAM OF EXPERIMENTAL SET UP SHOWING SECTIONS ANALYZED

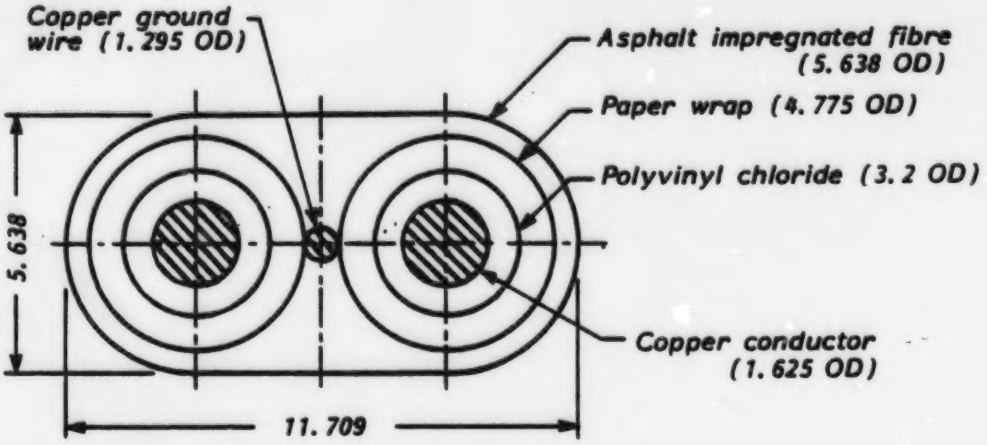
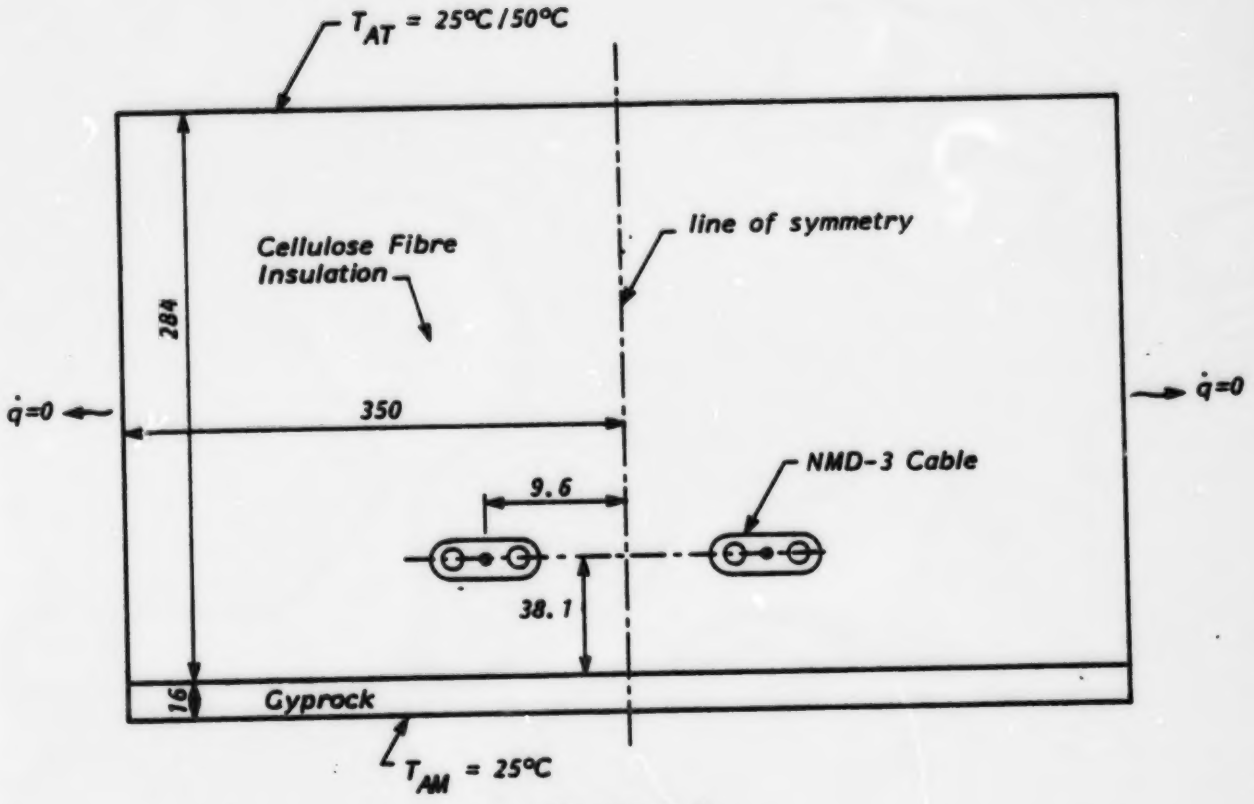


FIGURE 2
NMD-3 CABLE



Section A-A

(See Figure 1)

T_{AM} = Ambient Temperature

T_{AT} = Attic Temperature

All dimensions in mm

FIGURE 3

REGION ANALYZED FOR TWO-CABLE CONFIGURATION AND BOUNDARY CONDITIONS

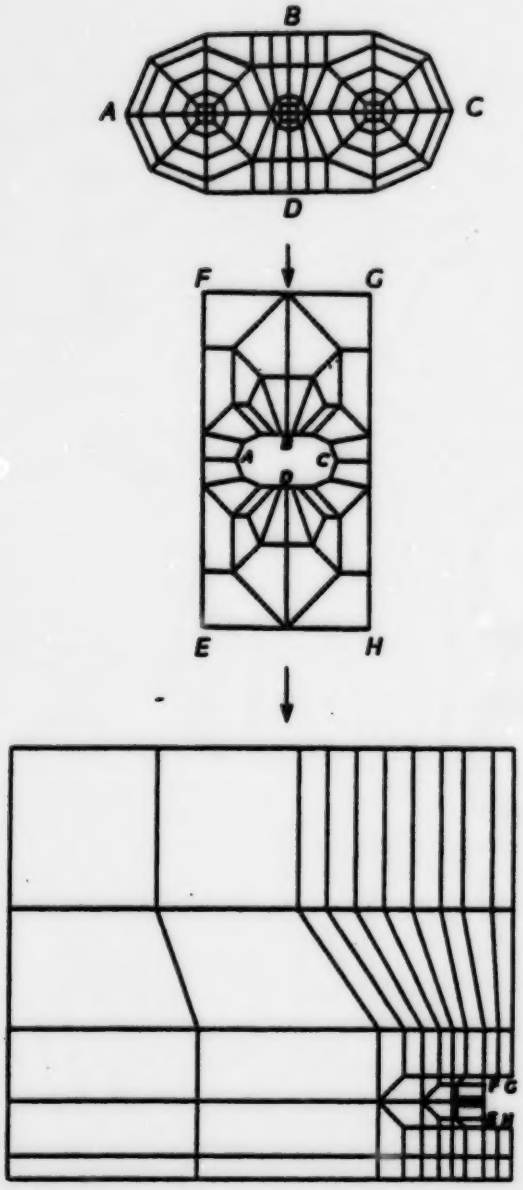
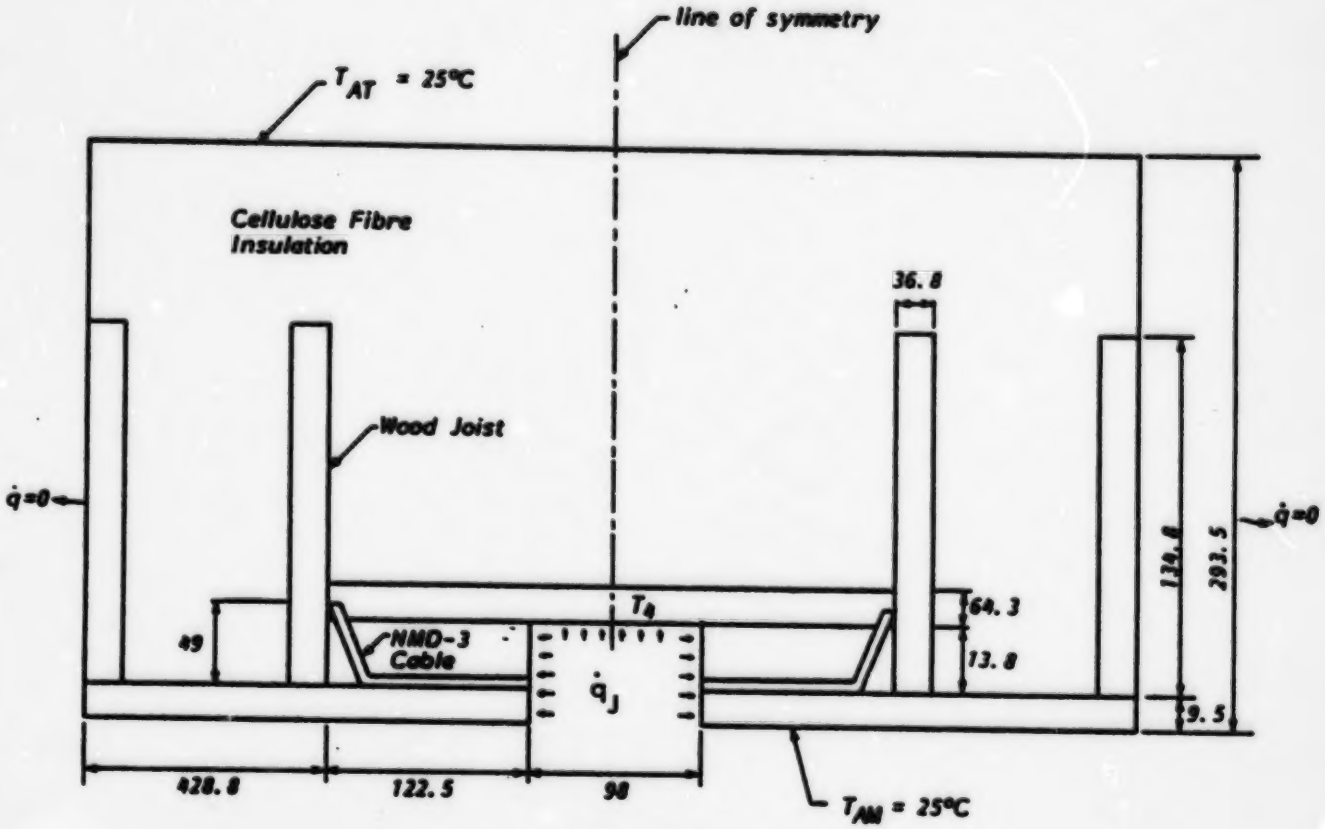


FIGURE 4

FINITE ELEMENT MODEL FOR TWO-CABLE CONFIGURATION



Section B-B

(See Figure 1)

T_{AM} = Ambient Temperature

T_{AT} = Attic Temperature

All dimensions in mm

FIGURE 5

REGION ANALYZED FOR ONE-CABLE CONFIGURATION AND BOUNDARY CONDITIONS

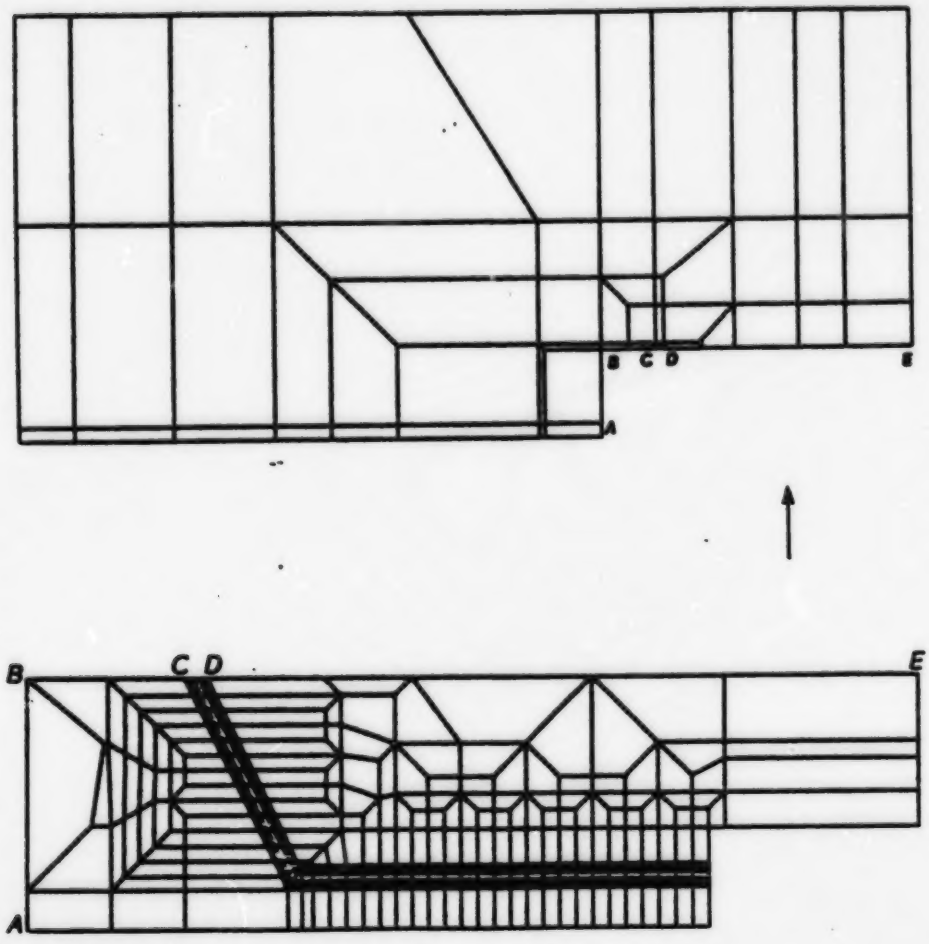
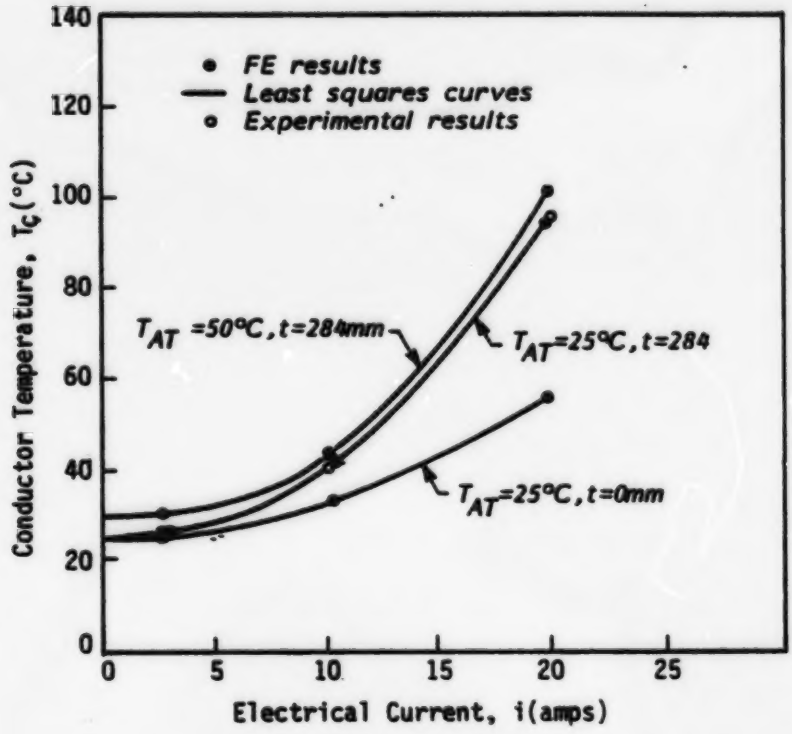


FIGURE 6
FINITE ELEMENT MODEL FOR ONE-CABLE CONFIGURATION

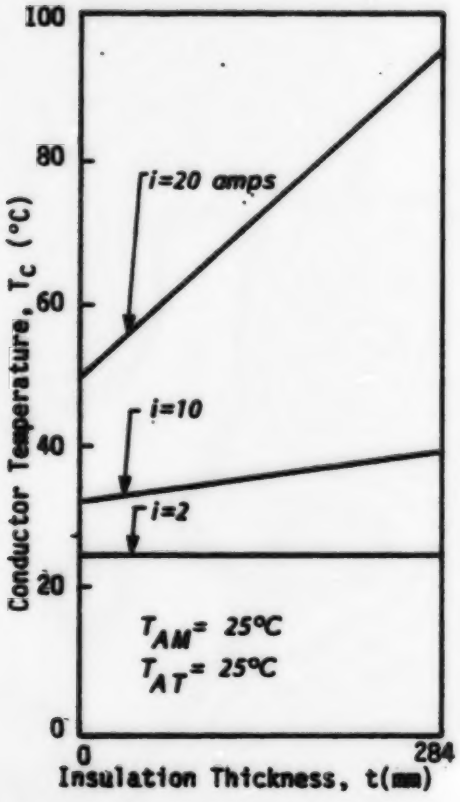


T_{AM} = Ambient Temperature

T_{AT} = Attic Temperature

FIGURE 7

CONDUCTOR TEMPERATURE AS A FUNCTION OF CURRENT WITH INSULATION THICKNESS AND ATTIC TEMPERATURE AS PARAMETERS



T_{AM}= Ambient Temperature

T_{AT}= Attic Temperature

FIGURE 8

CONDUCTOR TEMPERATURE AS A FUNCTION OF INSULATION THICKNESS WITH CURRENT AS A PARAMETER

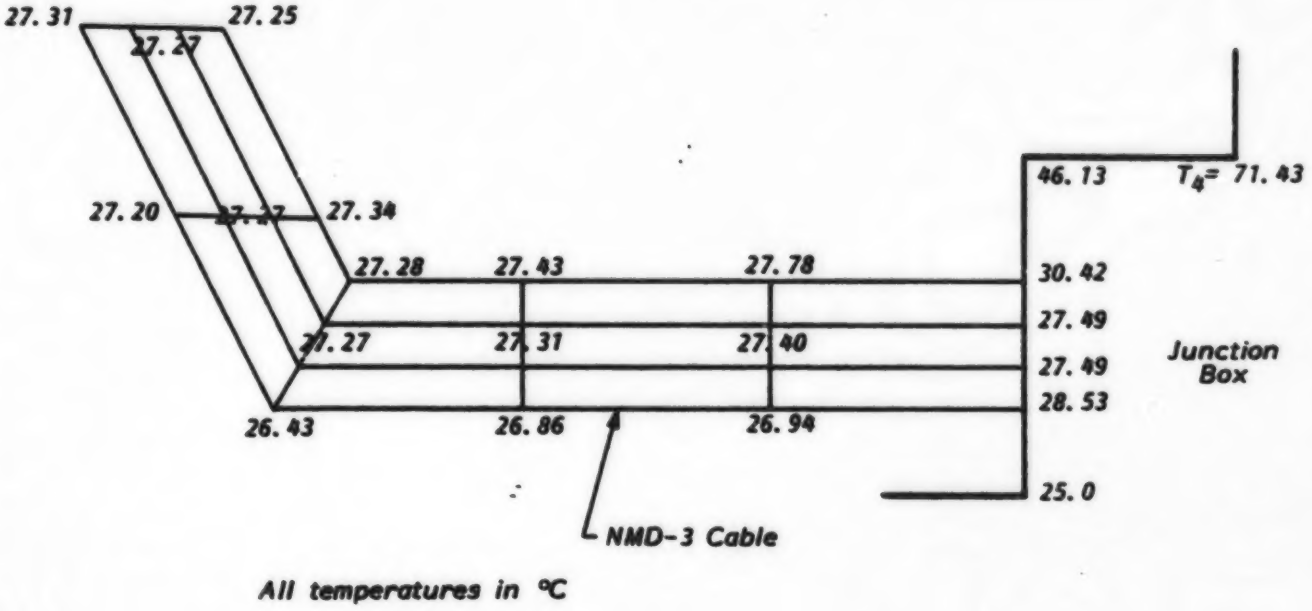


FIGURE 9

TEMPERATURE DISTRIBUTION ALONG THE CABLE FOR ONE-CABLE CONFIGURATION (i=2.7 AMPS)

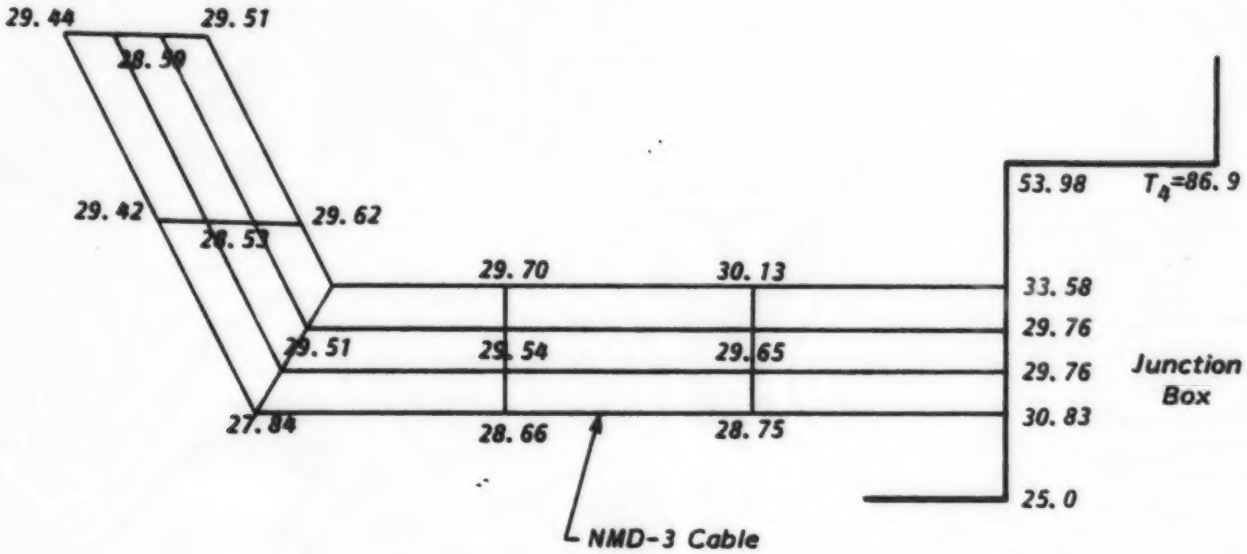


FIGURE 10

TEMPERATURE DISTRIBUTION ALONG THE CABLE FOR ONE-CABLE CONFIGURATION (i = 20 AMPS)

TABLE 1

THERMAL PROPERTIES

Material	*Thermal Conductivity (W/mm°C)*
Copper	372.10×10^{-3}
Polyvinyl Chloride	0.1315×10^{-3}
Paper	0.1881×10^{-3}
Asphalt	0.209×10^{-3}
Cellulose Fibre	0.08×10^{-3}
Gyprock	0.1874×10^{-3}
Wood	0.11076×10^{-3}

Note: Copper conductivity evaluated at 200°C.

All other conductivities evaluated at 25°C.

TABLE 2

CASES ANALYZED FOR TWO CABLES

Case No.	Insulation Thickness (mm)	Electrical Current (amps)	Attic Temperature (°C)
1	0 *	2	25
2		10	
3		20	
4	284	2	25
5		10	
6		20	
7	284	2	50
8		10	
9		20	

Ambient Temperature = 25°C

* Heat transfer from cable is by natural convection thus a heat transfer coefficient equal to 6.06×10^{-6} W/(mm^2. °C) has been used.

TABLE 3

CASES ANALYZED FOR ONE CABLE

Case No.	Insulation Thickness (mm)	Electrical Current (amps)	Attic Temperature (°C)	$\dot{q}_J$ (W/mm^2)	Exper/al T_4 (°C)	Theo/al T_4 (°C)
1	284	2.7	25	0.78×10^{-4}	71.8	71.4
2	284	20	25	1.06×10^{-4}	86.0	86.9

$\dot{q}_J$ = Total heat flux to the junction box

Ambient Temperature = 25°C

For T_4 location see Figure 1